ISSN 1068-3712, Russian Electrical Engineering, 2008, Vol. 79, No. 5, pp. 234–237. © Allerton Press, Inc., 2008.
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Application of Tensor Decomposition in Studying Linear Induction Motors

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Abstract—We consider questions of the development of mathematical and computer software for the analysis of electromagnetic processes in high-speed linear induction motors. It is shown that, upon applying tensor calculation methods to each part, it is possible to solve joint equations for the electric circuit, electromagnetic field, and motor control system.

DOI: 10.3103/S1068371208050027

The use of linear electric motors in public monorai systems makes it relatively simple to fit this transportation system into the existing urban infrastructure. Thrust being independent of the binding power between a wheel and the guideway, an electric drive with a linear motor is independent of weather conditions, which enables the mass of railway vehicles to be reduced. A linear induction motor (LID) appears to have considerable promise. The low production costs of LIDs may account for their use in traction drives of vehicles in the Moscow monorail transit system, which was the first public transportation system with linear motors in Russia. Here, the train performance features a low noise level and smoothness of movement. On account of the crookedness of the road, the average speed is low (of order 25–30 km/h) and does not exceed 50 km/h on tangent tracks, which is why a low-speed LID is applied here.

Experience in the application of electric drives on fully functional roads suggests considering projects for high-speed railways, which include lines connecting airports and cities, light metro projects, etc. The implementation of high-speed LIDs is more difficult due to the edge effects, which, unless proper measures are allowed for, materially reduce the performance of motors. It is known [1, 2] that, under a certain combination of parameters, the modular construction of an LID may improve its traction and energy performance, whereas the degree of improvement depends on the technical realization of such an implementation. In order to obtain a quantitative estimate of possible variants, a considerably complex problem has to be solved in connection with the joint calculation of the motor magnetic field and the calculation of currents in the motor windings under the variable-frequency voltage control of the motor power. In this paper, we solve this problem by using the mathematical apparatus of tensor diakoptics [3, 4].

In Fig. 1, we show several characteristic solutions for a two-module LID design.

Here, inductors are not only electrically connected, but they are also connected through a magnetic field, as the second inductor is positioned in the current loop magnetic field zone created by the first inductor in the secondary element (except for in the case in Fig. 1c). In the last case, the windings are concentric; the winding of the second module is nested within the first module winding. The windings are connected electrically in series.

For a one-dimensional model of the electromagnetic field in the inductor zone, the following equation [2] is used:

$$\frac{\delta'}{\mu_0}\frac{\partial^2 B}{\partial x^2} - \frac{a}{\rho}\frac{dB}{dt} - \frac{a}{\rho}v\frac{\partial B}{\partial x} = -\frac{\partial J_1}{\partial x}.$$

Upon applying the difference approximation, we have

$$\frac{2\delta'}{\mu_0}(B_+ - B) - \frac{4\delta'}{\mu_0}(B - B_-) - \frac{4ah^2}{\rho}\frac{\Delta B}{\Delta t}$$
$$-\frac{2ah}{\rho}V(B_+ - B_-) = -2h(J_+ - J_-).$$

We associate this difference equation with an electric circuit for the field area of length $2h$ (Fig. 2).

The following notation is adhered to in Fig. 2: J_1 is the linear density of an infinitely thin current layer of the inductor distributed throughout a segment of length $2p\tau$, where τ is the pole pitch; p is the number of pairs of the motor poles; ρ is the resistivity constant of the

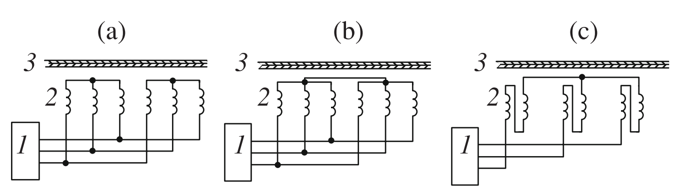


Fig. 1. Connection diagram of modules of linear induction. *1*—Voltage inverter. *2*—Inductor modules. *3*—Secondary element.

lining of a conducting material of thickness a, which is located on the secondary element: δ' is the nonmagnetic gap, including the lining thickness and the air space, which is characterized by the magnetic permeability μ_0; v is the secondary element speed; h is the approximation step based on central difference. The current source $2h(J_+ - J_-)$ is zero for a zone where there is no inductor. The voltages in the nodes of cell U_o, are numerically equal to the induction B at the approximating lattice nodes.

Depending on the computational accuracy, the full equivalent circuit of the calculated area may include anywhere from several hundred to several thousand elementary cells (shown in Fig. 2). Connections between nodal voltages of the circuit and current sources may be expressed by the formula

$$U_o = Z_{ov} J_v, \quad (1)$$

where Z_{ov} is determined from the equivalent circuit of the whole calculated area. Upon partitioning the entire circuit into separate subcircuits and applying tensor calculation methods by parts [4], it is possible to show that

$$Z_{ov} = (\tilde{Z}_{oo} - \tilde{Z}_{o\bar{o}} C_{\bar{o}s} (C_{s\bar{o}} \tilde{Z}_{\bar{o}\bar{o}} C_{\bar{o}s})^{-1} C_{s\bar{o}} \tilde{Z}_{\bar{o}o}) A_{ov} \quad (2)$$

where o are the subcircuits nodal coordinates, $\bar{o}$ are the nodal coordinates through which the subcircuits are connected between each other, s are the coordinates of the subcircuits couplings, v are the coordinates of branches. $\tilde{Z}_{o\bar{o}}$, is the reduced matrix of mutual resistances of nodal coordinates of subcircuits and nodal coordinates which connect the subcircuits, $C_{\bar{o}s}$ is the circuit matrix for transformation of subcircuits nodal coordinates into subcircuits coupling branches coordinates, A_{ov} is the nodal matrix of branches couplings in the subcircuit, $\tilde{Z}_{\bar{o}\bar{o}}$ is the matrix of reduced nodal resistances of the external nodes of subcircuits, and $\tilde{Z}_{oo}$ is the matrix of reduced nodal resistances of all nodes of subcircuits.

The current J_v of sources in separate cells in (1) is connected to the phase currents I_ϕ:

$$J_v = 1/(2h)\, C_{vp} C_{p\phi} I_\phi$$

where C_{vp} is the inductor branch-slot topological matrix and $C_{P\phi}$, is the winding matrix, connecting inductor slot currents with phase currents I_ϕ.

By making an allowance for (1), it is possible to connect the induction vector in the lattice nodes with the phase currents vector as follows:

$$U_o = 1/(2h)\, Z_{ov} C_{vp} C_{p\phi} I_\phi. \quad (3)$$

The integral of the induction over the area of the phase coils allows us to express the vector of phase EMFs E_ϕ in terms of the induction vector U_o. With the aid of matrix analysis, the numerical integration can be carried out by using the coil matrix $C_{\phi o}$:

$$E_\phi = j\omega H_i C_{\phi o} U_o = Z_e I_\phi.$$

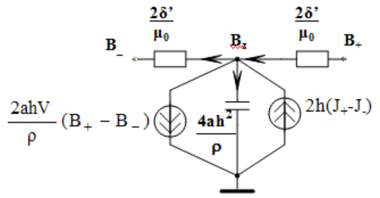


Fig. 2. Equivalent circuit of a field area with a length of 2*h*.

Here, $Z_e = j\omega H_i C_{\phi o} Z_{ov} C_{vp} C_{p\phi}$, is the inductor width and w is the power pulsation.

For phases voltages of the motor inductor, a matrix equation can be written as

$$U_\phi = R_\phi I_\phi + j\omega L_\sigma I_\phi + E_\phi$$
$$= (R_\phi + j\omega L_\sigma + Z_e) I_\phi = Z_U I_\phi, \quad (4)$$

where U_ϕ is the voltage vector applied to the inductor windings and R_ϕ and L_σ are the resistances matrix and the matrix of inductor windings stray inductances, respectively.

If we know the connection diagram of phase windings of the inductor, then the expression $U_\phi = Z_U I_\phi$ can be transformed to this connection diagram using the standard rules from coordinates tensor transformations. The connection of vectors and matrices in the contouring coordinates (designated by primed entities) with the phase coordinates is expressed by the following transformation formulas:

$$U'_\phi = C_{\phi t} U_\phi; \qquad Z'_U = C_{\phi t}\; Z_U\; C_\phi;$$
$$I_\phi = C_\phi\; I'_\phi; \qquad U'_\phi = Z'_U\; I'_\phi$$

Using the given vector of phase voltages, the current vector in the stator phases can be found by the formula

$$I_\phi = C_\phi\; (C_{\phi t} Z_U C_\phi)^{-1} C_{\phi t} U_\phi, \quad (5)$$

Expression (5) allows one to calculate the phase currents from the given voltage and simultaneously determine the values of the induction by formula (3). The topological matrices C_ϕ given in these expressions account for the connection of phase windings into the circuit.

The calculations by the formulas given were carried out using Fortran-95 programming language, the standard of which provides ample opportunities for matrices. The compiler g95 (http://www.g95.org) was supplemented by the Dislin library (http://www.dislin.de/) and the graphic package VFort (www.imamod.ru/~vab/) was used as a graphics-based environment.

The input data are divided into several groups. For a secondary element, the conductivity, width, and thickness are introduced. The frequency f_2, is necessary for calculating the motor power setting frequency while

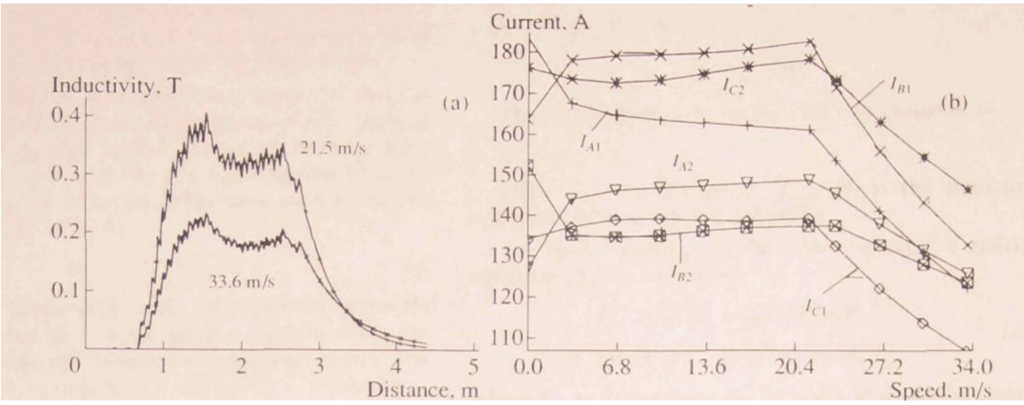


Fig. 3. Parallel connection of modules with insulated neutral: (a) is the induction distribution along the gapping at a speed of 21.5 and 33.6 m/s and (b) is the phase currents amplitude variation in the velocity function.

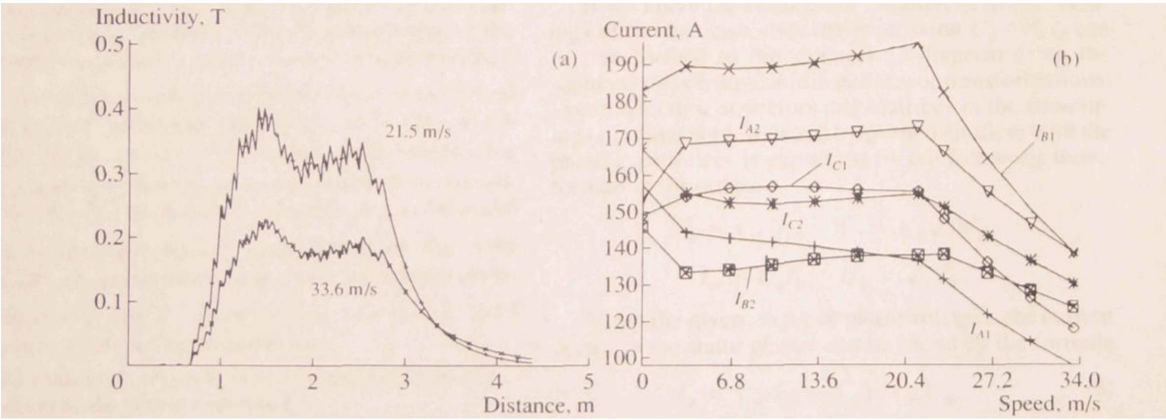
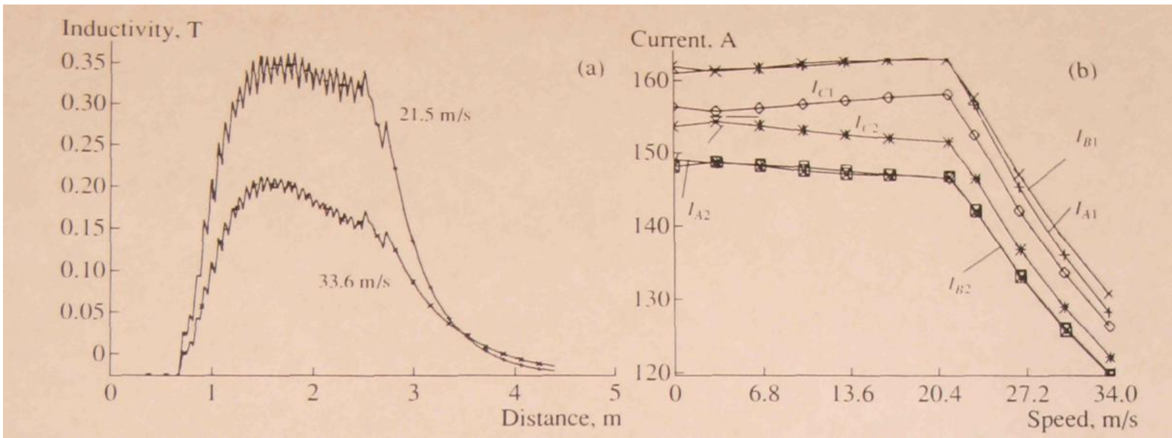


Fig. 4. Parallel connection of modules with common neutral: (a) is the induction distribution along the gapping at a speed of 21.5 and 33.6 m/s and (b) is the phase currents amplitude variation in the velocity function.

maintaining a constant absolute slip. In the program, the introduced coefficient of transversal edge biasing is multiplied by the conductivity of the secondary element in accordance to [5]. The air space includes the thickness of the secondary element lamination. The modules may be connected according to schemes from Fig. 1 except that, for these variants, it is possible to calculate the motor performances under the separate powering of modules. For the inductor, we introduce the resistance of the module phase, the module stray inductance, and the values of the amplitude and the voltage phase and/or the source current depending on the motor power setting. The winding matrix, which is necessary for calculations, can be formed automatically or introduced from a separate file, where the latter is used for complex windings with different structures on the edges of the inductor and at its center. For the automatic formation of the winding matrix, the number of conductor in the slot, the number of pairs of the poles, the phase sequence *(a-b-c, b-c-a,* or *c-a-b),* and the pole pitch are introduced. The setup of the calculation determines the integration method of the induction for calculating the phases EMF (three ways). Increasing the number of intervals in the partitioning of tooth spacing leads to an increase in the computational accuracy. The calculation was performed for the three following modes of the source: the first is a mixed mode in, until a change in speed occurs, the motor is fed by a current of constant amplitude at constant absolute slip, then a constant voltage excursion is maintained during an increase in the supply frequency that is proportional to the motor speed so that constant watt consumption condition is fulfilled; the second mode is the current source mode; and the third mode is the voltage source mode.

In Fig. 3, we show the calculation results for the parallel connection of an inductor with insulated neutral. In Fig. 4, we show the results for the parallel connection of inductors with common neutral, while in Fig.5



Fiy. 5. Series connection of modules of concentric windings: (a) is the induction distribution along the gapping at a speed of 21.5 and 33.6 m/s and (b) is the phase currents amplitude variation in the velocity function.

results are shown for the series connection of concentric windings.

As is clear from Figs. 3-5, the induction distribution along the gapping proves to be more uniform for concentric windings. For parallel connection of modules, the field pattern is nearly the same, which cannot be said about the distribution of phase currents over windings. The greatest dispersion of phase currents is observed in the scheme with common neutral. Despite the fact that the amplitude of the current source is constant, the phase currents vary greatly because of the edge effects. The knee in the characteristics in the graphs is caused by the switching of the motor power from a current source to a voltage source. The time for circuit design depends on the defined accuracy and varies anywhere from several seconds to several minutes.

CONCLUSIONS

1) The calculation method developed provides a means for the evaluation of various design concepts for the multimodule enclosure of a motor, in which the modules are connected, not only by an electric circuit, but also via a magnetic field.

(2) The calculations show that the application of concentric windings has proven to be the most useful technique. In this case, the distribution of the magnetic field over the inductor results in greater uniformity and the least dispersion of currents amplitudes in modules phases.

(3) The further development of this method is supports implementing the calculation of problems of high dimensionality, which calls for the improvement of calculation operations conducted for computational speedup.

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